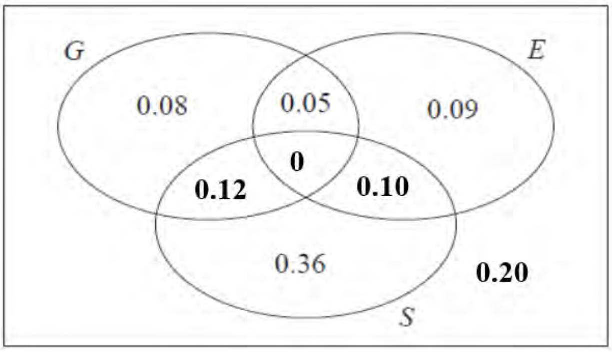
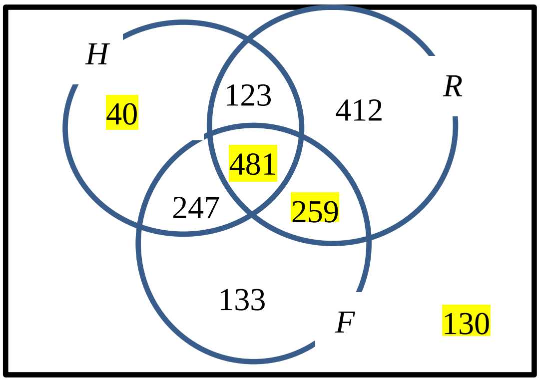
Qu 1	Scheme	Marks	AO
(a)	$0.08 + 0.09 + 0.36 = \underline{\mathbf{0.53}}$	B1	1.1b
		(1)	
(b)(i)	$\left[\mathrm{P}(G \cap E \cap S) = 0 \quad \Rightarrow\right]$ $\underline{\boldsymbol{p = 0}}$	B1	1.1b
(ii)	$[\mathrm{P}(G) = 0.25 \Rightarrow]\ 0.08 + 0.05 + q + "p" = 0.25$	M1	1.1b
	$\underline{\boldsymbol{q = 0.12}}$	A1	1.1b
		(3)	
(c)(i)	$\left[\mathrm{P}(S \mid E) = \frac{5}{12} \quad \Rightarrow\right] \frac{r + "p"}{r + "p" + 0.09 + 0.05} = \frac{5}{12}$	M1 A1ft	3.1a 1.1b
	$[12r = 5r + 5 \times 0.14 \quad \Rightarrow]$ $\underline{\boldsymbol{r = 0.10}}$	A1	1.1b
(ii)	$[0.08 + 0.05 + "0.12" + "0" + 0.09 + "0.10" + 0.36 + t = 1 \quad \Rightarrow]$ $\underline{\boldsymbol{t = 0.20}}$	B1ft	1.1b
		(4)	
(d)	$\mathrm{P}(S \cap E') = 0.36 +$ "q" $[= 0.48]$	B1ft	1.1b
	$\mathrm{P}([(S \cap E')] \cap G) = "q"[=0.12]$ **and** $\mathrm{P}(G) = 0.25$ **and** $\mathrm{P}(S \cap E') \times \mathrm{P}(G) = "0.48" \times \frac{1}{4}$ <u>or</u> 0.12	M1	2.1
	$\mathrm{P}(S \cap E') \times \mathrm{P}(G) = 0.12 = \mathrm{P}([(S \cap E')] \cap G)$ so **are independent**	A1	2.2a
		(3)	
		(11 marks)	

	Notes
(a)	B1 for 0.53 (or exact equivalent) [Allow 53%]
(b)(i)	B1 for $p = 0$ (may be placed in Venn diagram)
(ii)	M1 for a linear equation for q (ft letter "p" or their value if $0 \leqslant p \leqslant 0.12$) $\Rightarrow$ by $p + q = 0.12$
	A1 for $q = 0.12$ (may be placed in Venn diagram)
(c)(i)	M1 for a ratio of probabilities (r on num and den) (on LHS) with num < den **and** num <u>or</u> den correct ft. Allow ft of letter "p" <u>or</u> their p where $0 \leqslant p < 0.86$ but "+ 0" is not required.
	1st A1ft for a correct ratio of probabilities (on LHS) allowing ft of their p where $0 \leqslant p < 0.86$
	2nd A1 for $r = 0.1(0)$ or exact equivalent (may be in Venn diagram) **Ans only** 3/3
(ii)	B1ft for $t = 0.2(0)$ (o.e.) <u>or</u> correct ft i.e. $0.42 - (p + q + r)$ where p, q, r and t are all probs
(d)	B1ft for $\mathrm{P}(S \cap E') = 0.48$ (with label) (ft letter "q" or their value if $0 \leqslant q \leqslant 0.12$)
	M1 for attempting all required probs (labelled) <u>and</u> using them in a correct test (allow ft of q)
	A1 for **all probs correct** and a correct deduction (no ft deduction here)
SC	**No "P"** If correct argument seen apart from P for probability for all 3 marks, award (B0M1A1)
	If unsure about an attempt using conditional probabilities, please send to review.



Question	Scheme	Marks	AOs
2(a)	$\frac{365}{1825}$ or $\frac{1}{5}$ or 0.2 oe	B1	1.1b
		(1)	
(b)	$\frac{170}{1825}$ or $\frac{34}{365}$ or awrt 0.093	B1	1.1b
		(1)	
(c)	$90\times0.4+80\times0.05[=40]$ or $90\times0.6+80\times0.95[=130]$ or $740\times0.65[=481]$ or $740\times0.35[=259]$	M1	3.1b
		B1 B1 A1	1.1b 1.1b 1.1b
		(4)	
(d)	$P(R'\cap F)=\frac{380}{1825}\left[=\frac{76}{365}=0.208...\right]$ oe awrt 0.208	B1	1.1b
		(1)	
(e)	$\left[\frac{133\ +\ "130"}{1825}=\right]\frac{"263"}{1825}$ awrt 0.144	B1ft	1.1b
		(1)	
(f)	$\frac{247+"481"}{247+"481"+123+"40"}$	M1	3.4
	$=\frac{728}{891}$ awrt 0.817	A1	1.1b
		(2)	

Notes: **(10 marks)**

		Look out for answers given in the question. If you see answers in the question and in the answer space those in the answer space take precedence.
(a)	**B1**	Allow equivalent
(b)	**B1**	Allow equivalent
(c)	**M1**	Correct method to find one of the values 40 or 130 or 481or 259 Implied by 40, 481, 259 or130 seen in correct place on diagram
	B1	One of the highlighted correct
	B1	A second value highlighted correct or their $("259"+"\ 481")=740$ or their $("40"+"\ 481")=521$ or their $("40"+"130")=170$
	A1	Fully correct
(d)	**B1**	380/1825oe or awrt 0.208
(e)	**B1ft**	Correct answer or Ft their 130 (> 0) do not allow if blank Allow ft correct to 3 sf.
(f)	**M1**	For a single fraction with the numerator < denominator and *n* is an integer we will award for *n*/ 891 **or** *n*/(sum of their 4 values in *H*, each > 0) or awrt 0.817
	A1	728/891 oe or awrt 0.817

Qu 3	Scheme	Marks	AO
(a)	[0.13 + 0.25 =] **<u>0.38</u>**	B1	1.1b
		(1)	
(b)	Independence implies:		
	e.g. $\left[\mathrm{P}(B \cap C) = \mathrm{P}(B) \times \mathrm{P}(C) \quad \Rightarrow\right] \quad 0.3 = (0.3 + 0.05 + 0.25) \times (0.3 + p)$	M1	1.1b
	So $p = \underline{\mathbf{0.2}}$	A1	1.1b
	[Sum of probabilities = 1 gives] $q = \underline{\mathbf{0.07}}$	B1ft	1.1b
		(3)	
(c)	$[\mathrm{P}(A \mid B') =] \dfrac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.13}{(1-0.6) \text{ or } (0.13 + "0.2" + "0.07")}$	M1	1.1b
	$= \underline{\underline{\dfrac{13}{40}}}$ or **<u>0.325</u>**	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)	B1 for 0.38 (or exact equivalent)
	If answers are given on Venn Diagram <u>and</u> in the script then the script takes precedence.
(b)	M1 for a correct equation in p or P(C) only. May be implied by an answer of p = 0.2 **provided** this does not come from incorrect working. Condone missing brackets if they get 0.2 Other rules for independence will give simple rearrangements of this equation.
Beware	If p = 0.2 comes from incorrect working, we've seen $p = \frac{0.6}{0.3} = 0.2$, score M0A0
	A1 for p = 0.2 (or exact equivalent) B1ft for q = 0.07 (or exact equivalent) ft their p i.e. q = 0.27 – "0.2" where $0 \leqslant p \leqslant 0.27$
(c)	M1 for a correct ratio of probability expressions <u>or</u> a correct ratio of probabilities ft their values of p and q (provided both probabilities) <u>or</u> letters p and q A1 for 0.325 or exact equivalent. Correct answer only will score 2/2 **NB on epen this is labelled M1 but treat it as A1**

Qu 4	Scheme	Marks	AO
(a)	$\text{P}(S\cap\{X=50\})=\text{P}(S\cap\{X=80\})[=\text{a constant, } V]\Rightarrow b\times\frac{k}{50}=c\times\frac{k}{80}$	M1	3.1a
	May see: $\frac{k}{50}=\frac{V}{b}$ and $\frac{k}{80}=\frac{V}{c}$ (condone any letter for V even S)		
	So $c=\frac{8}{5}b$ *	A1cso*	1.1b
		(2)	
(b)	$d=2b$ **or** $a=\frac{2}{5}b$ **or** $c=4a$ **or** $d=5a$ **or** $d=\frac{5}{4}c$	M1 A1	2.1 3.3
	$\frac{2}{5}b+b+\frac{8}{5}b+2b=1$	M1	2.1
	$\Rightarrow 5b=1$ so $b=\frac{1}{5}$ (o.e.)	A1	1.1b
	$a=\frac{2}{25}$ $b=\frac{1}{5}$ $c=\frac{8}{25}$ $d=\frac{2}{5}$	A1	3.2a
		(5)	
(c)	[Experiment suggests for Nav] $\text{P}(S\mid\{X=100\})=0.3\Rightarrow k=30$ **or** $0.3=\frac{V}{0.4}\Rightarrow V=0.12$ So model won't work since $\text{P}(S\mid X=20)=\frac{30}{20}$ or $\frac{0.12}{0.08}$ and so would be greater than 1	B1	2.4
		(1)	
		(8 marks)	

	Notes
(a)	M1 for use of $\text{P}(S\mid X=x)\times\text{P}(X=x)$ for $x=50$ and $x=80$ (Must see k or their V) Any expression or equation MUST be based on the probability statements in qu.
*	A1cso for rearranging to required result, no incorrect work seen, condone poor notation
NB	Use of values e.g. $b=\frac{50}{20+50+80+100}$ to prove (a) is M0A0 but scores 2nd M1A1 in (b)
	Marks for (b) may be awarded for work seen in (a)
(b)	1st M1 for at least one other relationship (either probability the subject) from the list. 1st A1 for a second different relationship (either probability the subject) from the list. or Allow for: $\frac{ak}{20}=\frac{bk}{50}=\frac{ck}{80}=\frac{dk}{100}$ for 1st M1 1st A1 2nd M1 for using or stating sum of prob's = 1 May be implied by one correct probability. 2nd A1 for one correct probability e.g. $b=\frac{1}{5}$ or exact equivalent such as 0.2 3rd A1 for all correct probabilities. Allow exact equivalents e.g. $c=0.32$ **Sight of correct distribution or list of probs with no obvious incorrect working is 5/5**
(c)	B1 for deducing $k=30$ **and** giving a suitable example to show model breaks down

Notes on Question 4

The question essentially uses the definition of $\mathrm{P}(A \mid B)$ given in the formula booklet.

In particular $\mathrm{P}\left(S \mid \{X = x\}\right) = \dfrac{\mathrm{P}\left(S \cap \{X = x\}\right)}{\mathrm{P}\left(X = x\right)}$ $\boxed{1}$

The first "blob" tells us that $\mathrm{P}\left(S \mid \{X = x\}\right) = \dfrac{k}{x}$ where k is a constant.

The second "blob" tells us that $\mathrm{P}\left(S \cap \{X = x\}\right)$ is the same for all x so $\mathrm{P}\left(S \cap \{X = x\}\right) = V$ where V is a constant.

Using these results in $\boxed{1}$ gives $\dfrac{k}{x} = \dfrac{V}{\mathrm{P}(X = x)}$ $\boxed{2}$

Line 1 of MS for part (a) uses $V = \mathrm{P}\left(X = x\right) \times \dfrac{k}{x}$ for $x = 50$ and $x = 80$

Line 2 of MS for part (a) uses $\boxed{2}$ with $x = 50$ and $x = 80$

Other implications

Equation $\boxed{1}$ can be rearranged to give $\mathrm{P}\left(X = x\right) = x \times \dfrac{V}{k}$ $\boxed{3}$

So when $a + b + c + d = 1$ is used this gives $1 = \dfrac{V}{k}\left(20 + 50 + 80 + 100\right)$ or $\dfrac{V}{k} = \dfrac{1}{250}$ $\boxed{4}$

In particular if we use this relationship in $\boxed{3}$ the probabilities a, b, c and d can simply be written down for example $b = \dfrac{50}{250}$ as given in the **NB** in the notes on the MS.

The point is that k and V will vary according to equation $\boxed{4}$ but as part (c) shows there are some restrictions on the values k, and therefore V, can take.

Since $\dfrac{k}{x}$ is a probability then, ignoring the trivial cases*, $0 < \dfrac{k}{x} < 1$ and the "restricting" value of x is clearly $x = 20$ so $0 < k < 20$ and from $\boxed{4}$ we get $0 < V < \dfrac{20}{250} = \dfrac{2}{25} = a$

So the restrictions on k and on V are given by the shortest distance and its associated probability.

* $k = 0$ would say Tisam can never get the ball in the cup no matter what the distance.

$k = 20$ says she always gets the ball in the cup for any distance.

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